2020

COMPUTER APPLICATION

Course - 21

(Mathematics)

Full Marks : 50 | Time : Two Hours

The figures in the margin indicate full marks.

Answer Question No.1 and any *three* from the rest.

1. Answer any *two* of the following questions : 2½×2=5

   (a) State Laurent's Theorem.

   (b) Find Laplace transform of $\sin^2 at$.

   (c) Define conformal mapping.

   (d) Find the residence of $f(z)=\dfrac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z=3$

2. (a) Evaluate any *one* :

   (i) $\displaystyle\int \frac{dx}{\sin x(a+b\cos x)}$

   (ii) $L\left(te^{-t}\sin t\right)$ 5

   (b) Find the area bounded by the curves $y=x^2$ and $x=y^2$. 5

   (c) Prove that $\displaystyle\lim_{x\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\ldots+\frac{1}{2n}\right]=\log 2$. 5

3. (a) Expand $f(x)=x+x^2$ on $-\pi<x<\pi$. Hence deduce that

   $\dfrac{\pi^2}{6}=1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\ldots.\infty$ 10

   (b) Solve $\dfrac{dx}{dt}-7x+y=0$ 5

   $\dfrac{dy}{dt}-2x-5y=0$

4. (a) Use convolution theorem to find $L^{-1}\left\{\frac{p}{\left(p^2+4\right)^3}\right\}$ 5

(b) Evaluate any *one* : 5

(i) $L^{-1}\left\{\frac{3s+7}{s^2-2s-3}\right\}$

(ii) $L^{-1}\left\{\frac{p+1}{p^2+6p+25}\right\}$

(c) Using Laplace transform method, solve $\frac{d^2y}{dt^2}+y=t$, given that $\frac{dy}{dt}=1$, when $t=0$ and $y=0$ when $t=\pi$. 5

5. (a) Expand $\frac{1}{z\left(z^2-3z+2\right)}$ for 5

(i) $0<|z|<1$

(ii) $1<|z|<2$

(iii) $|z|<2$ 8

(b) (i) What are the Cauchy-Riemann conditions (or equations) for a function $f(z)=u+iv$ to be analytic ?

(ii) Show that C.R. equations are satisfied by the function $f(z)=u+iv$

where $f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}\,(z\neq 0)$

$f(0)=0$ 7

6. (a) Show that $\int_0^{\pi/2} \frac{dx}{\left(a^2\cos^2 x + b^2\sin^2 x\right)^2} = \frac{\pi}{4}, \frac{a^2+b^2}{a^3b^3};\ a,b>0$ 5

(b) Find $L^{-1}\left\{\frac{S-2}{(S-2)^2+5^2} + \frac{S+4}{(S+4)^2+9^2} + \frac{1}{(S+2)^2+3^2}\right\}$ 5

(c) Evaluate

(i) $\int \frac{dx}{a+b\sin x}$

(ii) $L\left(te^{-t}\sin t\right)$ 5
